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SIMULATION OF CASCADE PROCESSES IN TURBULENT FLOWS

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Model equations are derived for collective degrees of freedom, i.e. Fourier amplitudes of velocity field summated over the wave number octave (the wave number modulus changes twice within the octave). Stationary solutions of these equations which in the related inertial intervals yield the laws of similarity are analyzed ($k^{-5/3}$ in a three-dimensional turbulence and k^{-3} in a two-dimensional one). Nonstationary problems of forming cascade processes were numerically investigated in [1].

Simulation of cascade processes of energy transmission, vorticity, nonuniform concentration of admixtures is of particular interest in investigations of turbulent flows by the spectrum of turbulent motions. Cascade processes determine the inner structure of flows and the mechanism of turbulent dissipation. In the last few years it has been possible to simulate on a computer a two-dimensional space-periodic flow of not very high viscosity and to obtain a section of the energy spectrum $E(k) \sim k^{-3}$ [2-5] which corresponds to the cascade process of vorticity transfer [2, 6]. The authors are aware of only one publication [7] on numerical simulation of three-dimensional periodic flows, where the Reynolds numbers were not sufficiently high for the investigation of the cascade energy transmission process and obtaining a section of the spectrum governed by the "law of $^5/_3$".

Besides the refinement of numerical tests with a considerable number of degrees of freedom, it is expedient to develop methods for reducing the number of these without impending the realization of cascade processes. One of such methods is proposed below.

1. Model of cascade processes. Let us consider a space-periodic stream of incompressible viscous fluid. The Navier-Stokes equations for the Fourier components of the velocity field are of the form

$$\frac{\partial v_j(\mathbf{k})}{\partial t} = -i(\delta_{jl} - k_j k_l k^{-2}) k_m \sum_p v_l(\mathbf{p}) v_m(\mathbf{k}-\mathbf{p}) - \nu k^2 v_j(\mathbf{k}) + F_j(\mathbf{k}) \quad (1.1)$$

$$k_j v_j(\mathbf{k}) = 0, \qquad v_j^*(\mathbf{k}) = v_j(-\mathbf{k}) \quad (1.2)$$

where k_j are wave vector components whose values are $2\pi L^{-1} n_j$ (L is the space period and n_j are integers), δ_{jl} is the Kronecker delta, $F_j(\mathbf{k})$ are Fourier components of the external force field, and (1.2) define the conditions of solenoidality and reality of the velocity field, which are also satisfied by the field of forces $F_j(\mathbf{k})$. The twice repeated subscripts denote summation from unity to the number of measurements $s = 2, 3$. The term containing pressure is expressed in terms of velocity by using (1.2). The nonlinear term in the right-hand part of (1.1) defines the exchange of energy between motions of different scales. It presents considerable difficulties in numerical simulation. The quadratic character of nonlinearity has the effect of doubling the wave number, which is the essence of cascade processes. If at the initial instant of time the wave packet has $k \propto k_1$, then harmonics with $k \propto k_1 2^{i-1}$ $i = 2, 3, \ldots$ will subsequently appear.

Let us introduce Fourier amplitudes of the velocity field summated over the octave of wave numbers $\sqrt{2}k/2 \leqslant |\mathbf{k}'| \leqslant \sqrt{2}\,k$ (subsequently this is denoted by a zero superscript at the summation symbol)

$$u^2(k) = \left\langle \sum\nolimits^{\circ} v_j(\mathbf{k}') v_j(-\mathbf{k}') \right\rangle$$

where angle brackets denote the probable averaging over the set of realizations.

From (1.1) and (1.2) we have

$$u(k)\frac{\partial u(k)}{\partial t} = \Big\langle \sum\nolimits^{\circ} \Big[-ik_m' \sum_{\mathbf{p}} v_l(-\mathbf{k}') v_l(\mathbf{p}) v_m(\mathbf{k}'-\mathbf{p}) - \quad (1.3)$$

$$\nu(k')^2 v_l(\mathbf{k}') v_l(-\mathbf{k}') + F_l(\mathbf{k}') v_l(-\mathbf{k}') \Big] \Big\rangle$$

Since terms containing $p \propto k/2$ and $p \propto 2k$ play the main part in the formation of a cascade process, it is possible to approximate the first term in the right-hand part of (1.3) by the expression

$$\alpha_s k u(k) u^2(k/2) - \bar{\alpha}_s k u^2(k) u(2k)$$

where α_s, $\bar{\alpha}_s$ $(s = 2, 3)$ are dimensionless constants. The condition of conservation (for $\nu = 0$ and $F_j = 0$) of the quantity

$$I^{(m)} = \frac{1}{2} \sum_k k^m u^2(k) \quad (1.4)$$

where summation is for $k = k_1 2^{i-1}$, $i = 1, 2, \ldots$), yields

$$\bar{\alpha}_s^{(m)} = 2^{1+m} \alpha_s^{(m)}$$

The case of $m = 0$ relates to the conservation of energy $(s = 2, 3)$ and $m = 2$ to the conservation of the square of the vortex $(s = 2)$. Since in a three-dimensional flow energy is transferred from large to small scales, hence $\alpha_3^{(0)} > 0$. The same can be said about the transfer of vorticity in a two-dimensional flow, hence $\alpha_2^{(2)} > 0$. In a two-dimensional flow the energy is, however, transferred from small to large scales [6, 8]. Thus $\alpha_2^{(0)} < 0$.

Using notation

$$\left\langle \sum\nolimits^{\circ} (k')^2 v_l(\mathbf{k}') v_l(-\mathbf{k}') \right\rangle = \beta(k) k^2 u^2(k)$$

$$\left\langle \sum\nolimits^{\circ} F_l(\mathbf{k}') v_l(-\mathbf{k}') \right\rangle = u(k) F(k)$$

we obtain the following equation:

$$\frac{\partial u(k)}{\partial t} = \alpha_s^{(m)} k \left[u^2\left(\frac{k}{2}\right) - 2^{1+m} u(k) u(2k) \right] - \nu\beta(k) k^2 u(k) + F(k) \qquad (1.5)$$

If $h > 1$ is substituted for 2 as the module of scale changes and the summation in (1.4) is understood to be for $k_i = k_1 h^{i-1}$, $i = 1, 2, \ldots,$ then (1.5) changes to (*)

$$\frac{\partial u(k)}{\partial t} = \alpha_s^{(m)} k \left[u^2\left(\frac{k}{h}\right) - h^{1+m} u(k) u(kh) \right] - \nu\beta(k) k^2 u(k) + F(k) \qquad (1.6)$$

2. The stationary similarity mode. For a stabilized mode in the inertial interval of wave numbers (where the effect of viscosity and external energy sources is negligible) Eq. (1.6) assumes the form

$$u^2(kh^{-1}) = h^{1+m}\ u(k)\ u(kh) \qquad (2.1)$$

This equation has the solution

$$u(k) = A_s^{(m)} k^{-(1+m)/3} \qquad (2.2)$$

which corresponds to parametric similarity with the determining parameter

$$\varepsilon_s^{(m)} = \alpha_s^{(m)} k^{1+m} u^2(kh^{-1}) u(k) = \alpha_s^{(m)} (A_s^{(m)})^3 h^{2(1+m)/3} \qquad (2.3)$$

where $\varepsilon_s^{(0)}$ is the stream of energy over the spectrum $(s = 2, 3)$ and $\varepsilon_2^{(2)}$ is the stream of enstrophy (equal to half the mean square of the vortex).

According to Sect. 5 of Novikov's dissertation (see footnote) statistical characteristics of field $\mathbf{v}(\mathbf{k})$ in parametric similarity are invariant under transformations of the form

$$\lambda^{\sigma} \mathbf{v}(\lambda \mathbf{k}) \rightleftarrows \mathbf{v}(\mathbf{k})$$

where the arrows denote the statistical equivalence of fields, λ is an arbitrary number, and σ is the index of parametric similarity related to the dimension of the determining parameter χ by

$$\sigma = b\ /\ a, \qquad [\chi] = [v]^a\ [k]^b$$

*) This model of cascade processes was proposed by E. A. Novikov in a paper presented at the seminar of the L. D. Landau Institute of Theoretical Physics of the AS USSR in 1970 in connection with the theory of parameteric and scale similarity (see E.A.Novikov, Statistical models in the theory of turbulence. Doctoral dissertation, Moscow, 1969 and [9]).

For $\chi = \varepsilon_s^{(m)}$ from (2.3) we have

$$a = 3, \quad b = 1 + m, \qquad \sigma = (1 + m) / 3$$

From the relationship

$$u^2(k) = 2 \int_{kh^{-1/2}}^{kh^{1/2}} E(k_1)\, dk_1$$

and (2.2) and (2.3) for the spectral density of kinetic energy we obtain

$$E(k) = Bk^{-(5+2m)/3} \tag{2.4}$$

$$B = \frac{A^2 (1 + m)}{3 \left(h^{(1+m)/3} - h^{-(1+m)/3}\right)} = \frac{\varepsilon^{2/3} (1 + m)}{3\alpha^{2/3} h^{(1+m)/9} [h^{2(1+m)/3} - 1]} \tag{2.5}$$

where subscripts at B, A, ε, α are omitted.

In the considered here inertial interval of wave numbers it is possible to use the simpler relationship

$$E(k) \approx \frac{u^2(k)}{2\delta k} = \frac{A^2 h^{1/2}}{2(h-1)} k^{-(5+2m)/3}, \quad \delta k = k\left(h^{1/2} - h^{-1/2}\right) \tag{2.6}$$

which differs from (2.4) and (2.5) by a factor which for $h = 2$ and $m = 0,\ 2$ is close to unity.

For $m = 0$ Eqs. (2.4) and (2.5) yield the "law of $^5/_3$ " (for the reconciliation of the constant in the energy spectrum with experimental data it is necessary to set $\alpha_3^{(0)} \approx 0.2$) and for $m = 2$ the "law of 3".

Equation (2.1) admits the more general solution

$$u(k) = A' k^{-(1+m)/3} \exp\left[C (-2)^{\log_h k}\right] \tag{2.7}$$

which for $C \neq 0$ does not correspond to parametric similarity.

Numerical tests carried out by the authors [1] show that similarity is obtained in the considered model (with increasing number of cascades $C \to 0$). It is interesting to investigate this problem analytically particularly because such investigation explains singularities of the behavior of solutions for another cascade model considered here.

Let us consider a system consisting of n cascades, and write Eq. (1.6) in the finite-dimensional form

$$\frac{du_i}{dt} = \alpha_s^{(m)} k_i \left[u_{i-1}^2 - h^{1+m} u_i u_{i+1}\right] - \nu \beta_i k_i^2 u_i + F_i \tag{2.8}$$

$$u_i \equiv u(k_i), \ k_i = k_1 h^{i-1}, \ i = 1, \ldots, n; \ u_0 = u_{n+1} \equiv 0$$

The stationary mode of a cascade model is determined by the presence of sources and sinks of energy or vorticity. Let us assume, for simplicity's sake, that the external force acts only in the first cascade (*). This force is chosen so that amplitude u_1 which represents the external parameter of the problem remains constant. At reasonably high Reynolds numbers $R = \alpha u_1 k_1^{-1} \nu^{-1}$ (the estimate is given below) only the last step is affected by viscosity, hence it is reasonable to simplify the analysis by setting $\beta_i = 0$, $i = 1, \ldots, n - 1$ and $\beta_n = 1$. On these assumptions from (2.8) for the stationary state we have the system of equations

*) The case of $s = 2$, $m = 0$ and $\alpha < 0$, when energy is transmitted from small to large scales, can be considered in a similar way.

$$u_i^2 = h^{1+m} u_{i+1} u_{i+2}, \quad i = 1, \ldots, n-2 \tag{2.9}$$

$$u_{n-1}^2 = \nu \alpha^{-1} k_n u_n$$

We introduce the quantities

$$\varphi_i = \frac{1}{3}(i-1)\ (1+m) + \log_h (u_i u_1^{-1}) \tag{2.10}$$

which define deviations from the similarity mode. We obtain

$$2\varphi_i = \varphi_{i+1} + \varphi_{i+2}, \quad i = 1, \ldots n-2 \tag{2.11}$$

$$2\varphi_{n-1} = \gamma_n + \varphi_n, \ \varphi_1 = 0 \tag{2.12}$$

$$\gamma_n = (4+m)\ n / 3 - 2 - m - \log_h R \tag{2.13}$$

Setting $\varphi_i = \lambda^i$, from (2.11) we obtain

$$\lambda^2 + \lambda - 2 = 0, \quad \lambda_1 = 1, \quad \lambda_2 = -2$$

The proof that $\varphi_i = a + (-2)^i b$ yields the general solution of the equation for the recursion relation (2.11) is derived by the method of complete mathematical induction. The boundary conditions (2.12) yield

$$a = 2b = -\gamma_n [(-2)^n - 1]^{-1}$$

Finally, we obtain the unique stationary solution of the problem

$$u_i = u_1 h^{\varkappa}, \quad \varkappa = -\frac{(i-1)(m+1)}{3} + \gamma_n \frac{(-2)^{i-1} - 1}{(-2)^n - 1} \tag{2.14}$$

For $n \to \infty$ and $i / n < q < 1$ (q is a constant) solution (2.14) converts to the similarity mode (2.2) with a constant independent of R and determined by (2.3). If R is considered as a function of n, then by virtue of (2.13) for the indicated transition to limit it is sufficient to specify that the increase of $\log_h R$ with increasing n must be slower than exponential. Such requirement is entirely justified, since for the simulation of a turbulent flow it is reasonable to choose the number n of steps on the basis of the requirement

$$k_n \equiv k_1 h^{n-1} \gtrsim (l_\nu^{(m)})^{-1} = (\varepsilon^{(m)} / \nu^{-3})^{1/(4+m)} \approx k_1 R^{3/(4+m)} \tag{2.15}$$

where $l_\nu^{(m)}$ is the inner scale of turbulence which for $m = 0$ is the same as the Kolmogorov inner scale [10], and for $m = 2$ as the scale introduced in [2, 6]. If equality is substituted for the inequality in (2.15), then for maximum R we obtain

$$\log_h R \approx {}^1/_3 (n-1)\ (4+m) \tag{2.16}$$

In that case the effect of viscosity will make itself felt only in the last step, and γ_n becomes independent of n.

The similarity mode for the considered model is in a certain sense stable with respect to small-scale perturbations. Disregarding the specific properties of the boundary conditions (2.12), from (2.11) we directly obtain

$$\delta\varphi_{i-l} = (-{}^1/_2)^l\, \delta\varphi_i, \qquad \delta\varphi_i = \varphi_{i+1} - \varphi_i \tag{2.17}$$

Thus the deviation from the similarity law in the region of high wave numbers (e.g., owing to the viscosity effect) is rapidly attenuated with increasing penetration into the region of smaller wave numbers.

3. Obukhov's model. In analyzing the system of equations of the hydrodynamic kind, Obukhov has investigated in detail the case of the triplet [11]. He considered a system of linked triplets and derived a definite model for the cascade process [12]. According to the proposed here classification Obukhov had considered the case of $m = 0$ and $\alpha > 0$ ($s = 3$). Equations of Obukhov's model, when extended to the case of any arbitrary m, are in our notation of the form

$$\begin{gathered} \partial u_i/\partial t = \alpha k_i\,[u_{i-1}u_i - h^{1+m}u_{i+1}^2] - \nu\beta_i k_i^2 u_i + F_i \\ k_i = k_1 h^{i-1}, \quad h > 1, \quad i = 1, \ldots, n; \quad u_0 = u_{n+1} \equiv 0 \end{gathered} \tag{3.1}$$

At first glance Eqs. (2.8) and (3.1), derived independently on different considerations, are very similar. In the absence of viscosity and external forces Eq. (3.1), as well as (2.8), admit a stationary solution which corresponds to the similarity mode (2.2). However Eq. (3.1) admits, also, a stationary solution for $\beta_i \equiv 0$ and $F_i \equiv 0$, in which the entire energy is concentrated in the first harmonic ($u_1 \neq 0$ and $u_i = 0$, with $i > 1$), while (2.8) has no such solution. Moreover, when external forces do not directly act on the small-scale harmonics ($F_i \equiv 0,\ i > l$) , and these harmonics are not excited at the initial instant of time ($u_i(0) = 0, i > l$), then in accordance with (3.1) the energy is not transmitted from large-scale harmonics to small-scale harmonics. On the other hand, in the model (2.8) small-scale harmonics are excited by large-scale ones. Below we present the analysis which is similar to that given in Sect. 2 for that model.

If in the stationary mode the external force maintains u_1 constant and viscosity affects only the last step (on the assumption that $u_n \neq 0$; for $u_n = 0$ and $\beta_{n-1} = 1$ the problem reduces to that considered in Sect. 2 for model (2.8) by substituting $n - 1$ for n) we have

$$\begin{gathered} u_i u_{i+1} = h^{1+m} u_{i+2}, \quad i = 1, \ldots, n-2 \\ u_{n-1} = \nu\alpha^{-1}k_n \end{gathered} \tag{3.2}$$

Introducing φ_i defined by (2.10), we obtain

$$\varphi_i + \varphi_{i+1} = 2\varphi_{i+2}, \quad i = 1, \ldots, n-2 \tag{3.3}$$

$$\varphi_1 = 0, \quad \varphi_{n-1} = \gamma_n' \tag{3.4}$$

$$\gamma_n' = n\,(4+m)/3 - (5+2m)/3 - \log_h R \tag{3.5}$$

The general solution of (3.3) is of the form

$$\varphi_i = a_1 + \left(-\frac{1}{2}\right)^i b_1$$

Having determined a_1 and b_1 by the boundary conditions (3.4), we finally obtain

$$u_i = u_1 h^{\varkappa_1}, \quad \varkappa_1 = -\frac{(i-1)(m+1)}{3} + \gamma_n' \frac{1-(-1/2)^{i-1}}{1-(-1/2)^{n-2}} \tag{3.6}$$

Note that Eqs. (3.2) and their extension to the case in which all $\beta_i \neq 0$ have a second solution which differs by the sign of u_n. For $n \to \infty$ and $\gamma_n' \neq 0$ solution (3.6) does not convert to the similarity mode. For $i \to \infty$ (3.6) formally yields a regular power dependence on the wave number, but the proportionality coefficient depends then substantially on viscosity.

From (3.3) for $\delta\varphi_i$ we obtain

$$\delta\varphi_{i-l} = (-2)^l \delta\varphi_i$$

Unlike in the case of (2.17), here the small-scale deviations from the stationary similarity mode increase with penetration into the region of large scales (small wave numbers). It further appears that the stationary solution for model (3.1) in a linear approximation is unstable, since the trace of the matrix resulting from such approximation is positive [13]. A. B. Glukhovskii and A. B. Karunin came to the same conclusion about the instability of the stationary solution for model (3.1).

Numerical tests carried out by the authors show that the rapidly developing perturbations in model (3.1) move it away from the similarity mode.

4. Formal conclusion and generalization of models. Neglecting external forces and dissociation, we shall show how models (2.8) and (3.1) and their generalization are derived on the basis of the following general requirements: (1) quadratic properties of nonlinear terms; (2) scale invariance of dimensionless coefficients in the equation; (3) direct interaction only between closest neighbors in the spectrum (an approximation widely used in theoretical physics); (4) presence of the quadratic integral (1.4).

It follows from requirements (1)–(3) that

$$\frac{\partial u(k)}{\partial t} = k\,[a_1u^2(kh^{-1}) + a_2u(kh^{-1})\,u(k) + a_3u(kh^{-1})\,u(kh) + a_4u^2(k) + a_5u(k)\,u(kh) + a_6u^2(kh)]$$

where $a_1, \ldots, a_6$ are dimensionless coefficients independent of k.

Condition (4) yields

$$a_3 = a_4 = 0, \quad a_5 = -h^{1+m}a_1, \quad a_6 = -h^{1+m}a_2$$

Finally we obtain

$$\frac{\partial u(k)}{\partial t} = a_1k\,[u^2(kh^{-1}) - h^{1+m}u(k)\,u(kh)] + a_2k\,[u(kh^{-1})\,u(k) - h^{1+m}u^2(kh)] \tag{4.1}$$

The derived equation has the stationary solution (2.2) which corresponds to the similarity mode. For $a_2 = 0$ from (4.1) we obtain model (1.6), (2.8), and for $a_1 = 0$ model (3.1). The first model converts to the second for the transformation

$$h \to h^{-1}, \qquad t \to -h^{1+m}t$$

i.e. for the reflection of scale change and of time. Neglecting the limitation (3) it becomes possible to obtain a more general class of models which are considered in [14].

To this more general class belongs, in particular, the model recently described in [15]. However numerical tests carried out by the authors show that in this model similarity modes do not obtain.

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DETERMINATION BY TEST PUMPING OF VARIABLE PERMEABILITY OF A STRATUM UNDER CONDITIONS OF RADIAL SYMMETRY

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A central borehole in a circular stratum whose permeability depends only on the radius is considered. The permeability coefficient is determined by the flow rate and the pressure in the borehole with constant pressure at the contour of the latter. The problem of determination of the permeability coefficient reduces to the restitution of the Sturm-Liouville operator over its spectral function. Nominal correctness of the problem is proved in the case in which the permeability coefficient belongs to the class of bounded positive functions with bounded first and second derivatives.